

The Role of VLBI in the Weekly Inter-Technique Combination

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Abstract

The geodetic space-techniques VLBI, GPS, and SLR show a different grade of sensitivity to geodetic and geophysical parameters such as station coordinates and Earth orientation parameters (EOP). While the satellite techniques GPS and SLR are, in principle, able to determine the Earth's center of mass, VLBI is, as a unique technique, able to determine all EOP such as the motion of the celestial and the terrestrial pole and the rotation angle of the Earth. The accuracy of all obtained parameters depends strongly on the network geometry of the technique-specific observing stations. To benefit from the strength of each geodetic space-technique and to get the most stable solution of station coordinates and all EOP within one adjustment, the techniques could be combined in an inter-technique combination. Within the common approach of combining multi-year technique-specific normal equations, the weights of the techniques are assumed to be constant over time. In this paper, weekly combined solutions are presented. The weighting is realized by a variance component estimation (VCE). The obtained relative weights are not constant over time. The VCs of the 'IVS-R1' sessions show a clear seasonal variation which is not yet fully understood. Nevertheless, an improvement of the EOP due to a session-wise VCE-based weighting could be achieved in some cases compared to a constant weighted solution.

1. Introduction

The geodetic space techniques GPS, SLR and VLBI show a different sensitivity to geodetic and geophysical parameters such as the station coordinates, the Earth orientation parameters (EOP), or the Earth's gravity field coefficients. The satellite techniques GPS and SLR are both able to determine the Earth's center of mass, but they are only sensitive to the rates of change of the Earth rotation angle UT1-UTC and the celestial pole coordinates X and Y [2]. The outstanding ability of VLBI to determine all EOP (celestial and terrestrial pole, UT1-UTC) in an absolute sense makes a combination of the different techniques very reasonable. Up to now, the combination of the techniques for computing terrestrial reference frames has been based on technique-specific normal equations (NEQs) containing all observations of one technique (multi-year reference frame (MRF)). Therein, the station motion is parameterized with a position at a reference epoch t_0 and a constant velocity. Hence, the relative weighting of the NEQs is assumed to be constant over time. In this paper, weekly combined solutions are computed. The weighting can be done individually for each week (session) or constantly over time. A variable weighting has the advantage that differences in the quality inherent in the input data can be taken into account. In this paper, the variable weighting of the techniques is done using a variance component estimation (VCE) algorithm [1].

2. Processing Algorithm

The epoch-wise combination of the different techniques is done at DGFI at the normal equation level. The normal equation matrices of the geodetic space techniques GPS, SLR, and VLBI are added to a weekly combined normal equation matrix. A simplified flow chart of the combination

process is given in Figure 1. An important step in the combination is the relative weighting of the NEQs. Since the weights are not assumed to be constant over time, for every weekly NEQ an iterative estimation of the relative weights using a VCE is performed.

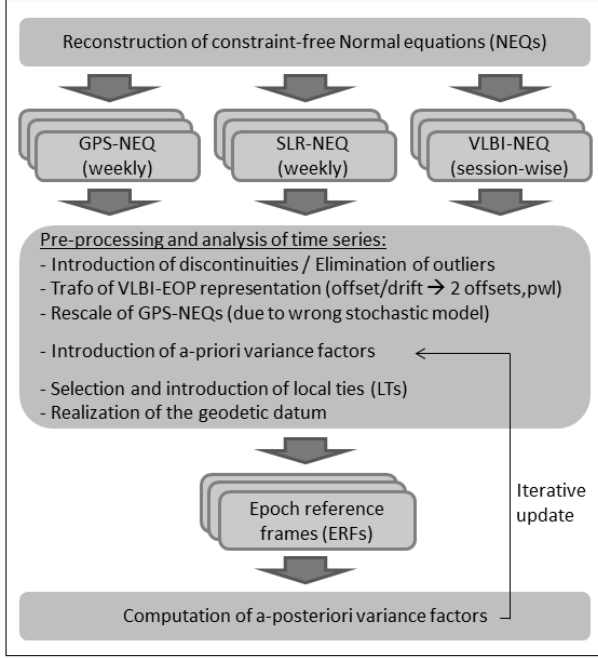


Figure 1. Simplified flow chart of the epoch combination approach.

\mathbf{N}_i and \mathbf{y}_i are the normal equation matrix and the corresponding right hand side of the i -th individual equation system. m_i is the number of observations (including the number of pre-reduced parameters). The matrix \mathbf{N}_c is the weighted sum of the individual matrices \mathbf{N}_i according to

$$\mathbf{N}_c^{(k)} = \sum_i \frac{1}{\sigma_i^{2(k)}} \mathbf{N}_i. \quad (4)$$

The matrix $\mathbf{l}_i^T \mathbf{P}_i \mathbf{l}_i$ is the weighted sum of the observations squared. In the combination, one VC for each weekly GPS- and SLR-NEQ is estimated. For VLBI, one VC for each session NEQ is estimated. At the end, time series of combined station coordinates, EOP, and VCs for each technique are obtained.

3. Variance Components

Figure 2 shows the time series of VCs for GPS, SLR, and VLBI between 1994.0 and 2007.0. As weights for the NEQs, the reciprocal values of the VCs are used (Equation (4)). Therefore, a low VC means a high weight in the combination process. For GPS, all VCs are nearly equal to 1.0 except the VCs between 1994.0 and 1996.0. This might be due to the fact that within this time period, less globally well-distributed stations were available. Hence, the accuracy of these GPS NEQs w.r.t. the NEQs after 1998.0 is decreased, and their impact on the combined NEQs varies

The a posteriori variance component (VC) of the i -th individual normal equation matrix of the $(k+1)$ -th iteration step is computed according to

$$\sigma_i^{2(k+1)} = \frac{\Omega_{c,i}^{(k)}}{r_{c,i}^{(k)}} \quad (1)$$

with the weighted sum of the residuals squared

$$\Omega_{c,i}^{(k)} = \hat{\mathbf{x}}_c^{(k)T} \mathbf{N}_i \hat{\mathbf{x}}_c^{(k)} - 2 \mathbf{y}_i^T \hat{\mathbf{x}}_c^{(k)} + \mathbf{l}_i^T \mathbf{P}_i \mathbf{l}_i \quad (2)$$

and the partial redundancy (degree of freedom)

$$r_{c,i}^{(k)} = m_i - \frac{1}{\sigma_i^{2(k)}} \text{tr}(\mathbf{N}_i \mathbf{N}_c^{(k)-1}). \quad (3)$$

$\hat{\mathbf{x}}_c^{(k)}$ is the vector of the estimated parameters in the (k) -th iteration step.

(by about 5%). After 1998.0, a small seasonal variation of the GPS VCs is visible. The mean VC for SLR before 2000.0 is 6.12. After this epoch, it decreases to a mean value of 2.34 (see also Table 1). This decrease by about 62% is explained with the improvement of the SLR observation network since 1994.0. The VLBI VCs can be allocated to the particular session types. Between 1994.0 and 1998.0, a bulge of the VCs occurs. This bulge seems to coincide with the signature of the GPS VCs at this time. Although the VLBI sessions ‘NEOS’, ‘CORE’, and ‘IRIS’ are scheduled in order to determine accurate EOP measurements (therefore, a good station distribution is used and the VCs of these session types are expected to be near 1), the VCs of these sessions are much larger than 1 (Table 1). The large VCs are dominated by the bulge in the VCs before 1998.0 which is caused by the combination.

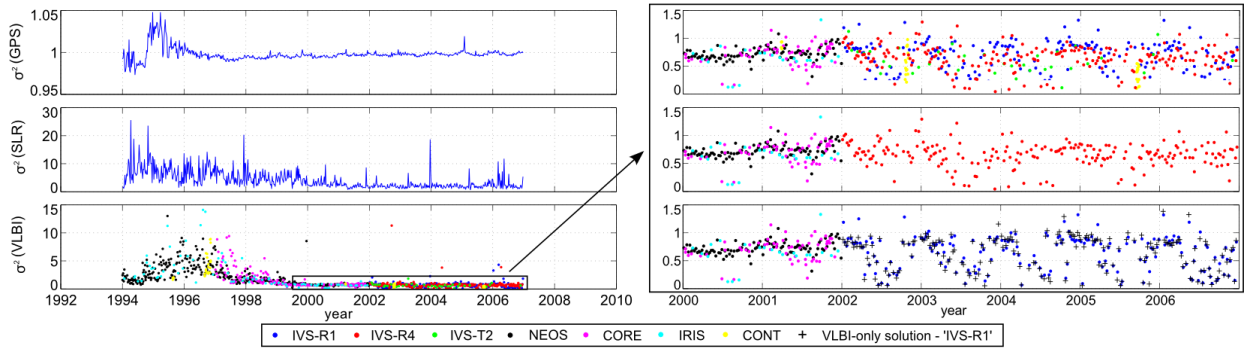


Figure 2. Left plots: a posteriori VCs of the techniques of GPS (upper), SLR (middle), and VLBI (lower). In the case of VLBI, one VC per session is estimated. Right plots: zoom of VLBI plot for 2000.0 — 2007.0. All VLBI sessions are shown for 2000 and 2001. The data shown for 2002.0 — 2007.0 is all VLBI sessions (upper), IVS-R4 sessions only (middle), and IVS-R1 sessions only (lower). In addition, the lower right plot adds the a posteriori VCs of the VLBI-only solutions for the IVS-R1 sessions for comparison.

The right part of Figure 2 shows the VLBI variance components between 2000.0 and 2007.0. A periodic variation of the VCs is visible since 2002.0. If the components of the session types ‘IVS-R1’ and ‘IVS-R4’ (which have been scheduled by the IVS since 2002.0) are separated, it is clearly visible that only the ‘IVS-R1’ sessions show an annual variation with a minimum in the summer. This means that the impact of the VLBI NEQ on the combined NEQ during the summer is higher than during the winter time (Equation 4). As described above, also the GPS VCs show a small annual variation after the epoch 2002.0 which is in phase with the VLBI variation. In order to find the technique responsible for this variation, the a posteriori variance factors of the VLBI-only ‘IVS-R1’ solutions are shown additionally to the VLBI VCs of the VCE in the lower right plot of Figure 2. The annual variation occurs not only in the VCs of the VCE but also in the variance factors of the single technique solutions. This proves that the annual variation is caused by VLBI. The reason for the periodic behavior of the VLBI VCs is not fully explained yet.

4. Earth Orientation Parameter

One type of estimated parameters in the combined solutions is EOP. All EOP are parameterized as a piecewise linear segmented line with estimated offsets at the midnight epochs. Per weekly NEQ, eight EOP offsets are included. Since the satellite techniques are only sensitive to the rates

Table 1. Mean VCs for GPS, SLR, and each VLBI session type before and after the epoch 2000.0.

solution	$t < 2000.0$	$t \geq 2000.0$	solution	$t < 2000.0$	$t \geq 2000.0$
GPS	1.0	1.0	IVS-T2	—	0.64
SLR	6.12	2.34	NEOS	2.199	—
VLBI (all sessions)	2.63	0.91	CORE	1.633	—
IVS-R1	—	0.70	IRIS	3.547	—
IVS-R4	—	0.73	CONT	2.123	0.926

of change of UT1-UTC and the celestial pole coordinates (X,Y), at least one offset of the estimated segmented line has to be fixed to its a priori value in order to repair the rank deficiency of the NEQ. The other midnight offsets are extrapolated using the rates. Since the rates are highly correlated with the orbit parameters and consequentially are affected by orbit systematics, the extrapolated offsets show a systematic deflection w.r.t. the reference time series IERS 08 C04 (see right plot of Figure 3). If the NEQs of the satellite techniques are combined with a VLBI NEQ, which contains

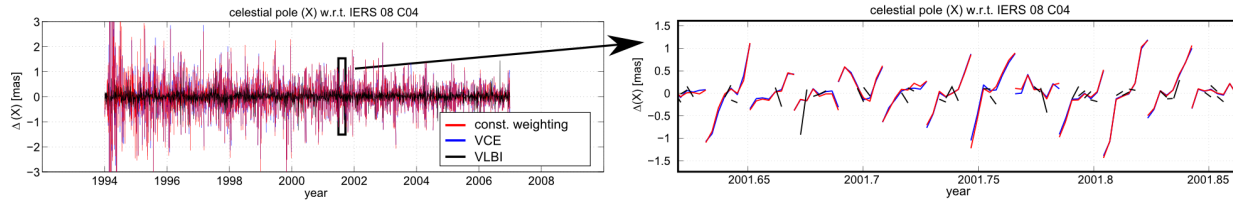


Figure 3. Estimated celestial pole coordinates in x-direction w.r.t. the IERS 08 C04 time series for the VLBI-only solution and the combined solution with and without variable weighting of the techniques.

absolute information about the offsets, the constraints are not necessary any longer. Usually, at least two VLBI sessions are scheduled during one week (‘IVS-R1’ on Monday and ‘IVS-R4’ on Thursday). The offsets in the combined solution in between the VLBI epochs are extrapolated with the rates delivered by the satellite techniques. As follows, the segmented line in between the VLBI epochs shows systematic differences w.r.t. the reference time series (Figure 3).

In Table 2, the weighted mean RMS values of the EOP of the single technique solutions and of the two combined solutions (constant weighted and VCE-based weighted) are given. In the case of the celestial pole coordinates and UT1-UTC, the constant weighted combination shows the largest scatter. If a VCE is used, the scattering decreases slightly. The large scatter is explained by the deflected parts of the segmented line in between two VLBI epochs. If only the epochs with three techniques contributing are considered (VCE-based solution at VLBI epochs), the WRMS values decrease significantly but still are larger than the WRMS values for the VLBI-only solution (celestial pole: 5 to 7%, UT1-UTC: 45%). In the case of the terrestrial pole coordinates, the constant weighted combination shows larger scatter than the GPS only solution. In contrast to this, if a VCE is used, the scattering is at the level of the GPS-only solution (slight improvement of the x-coordinate and slight degradation for the y-coordinate).

Table 2. Weighted mean RMS values of the terrestrial and celestial pole coordinates and UT1-UTC w.r.t. the IERS 08 C04 time series for the GPS-only, the VLBI-only, and different combined solutions.

WRMS	GPS	VLBI	const. weighted	VCE	VCE (VLBI epochs)
cel. pole (X) [μas]	—	88.7	240.9	239.8	94.8
cel. pole (Y) [μas]	—	95.3	112.5	112.4	100.0
UT1-UTC [μs]	—	12.1	39.9	39.5	17.5
terr. pole (x) [μas]	123.0	213.7	142.3	122.7	109.8
terr. pole (y) [μas]	114.2	248.2	136.6	117.9	107.5

5. Conclusions

Within the inter-technique combination, VLBI plays a central role. It is the unique technique to determine the absolute offsets of UT1-UTC and the celestial pole coordinates. Therefore, no constraints for these parameters for the satellite techniques are necessary in the combination. The results have shown that the combination using a VCE-based weighting allows consideration of quality differences inherent in the input data. If a variable weighting of the techniques using a VCE is realized, the weights of the VLBI NEQs w.r.t. the other NEQs in the combination show some systematics, which have to be further investigated. Since 2002.0, the VCs of the ‘IVS-R1’ sessions have shown an annual variation between 0 and 1 with a minimum in the summer. This variation also occurs in the a posteriori VCs of the VLBI-only solutions. It verifies that the reason for this variation is caused by the VLBI technique.

The estimates of the UT1-UTC and the celestial pole coordinates of the combined solutions, where a VCE is used for the weighting, and only VLBI epochs are considered, are comparable to those of the VLBI-only solutions. In the case of the terrestrial pole coordinates, GPS provides the most stable parameter time series w.r.t. the IERS 08 C04 time series. The weighted scattering of the estimates of the combined solutions using a VCE are here at the same level as the GPS-only solutions (differences below 3%).

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